BAMBOO: Accelerating Closed Itemset Mining by Deeply Pushing the Length-Decreasing Support Constraint*

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Abstract

Previous study has shown that mining frequent patterns with length-decreasing support constraint is very helpful in removing some uninteresting patterns based on the observation that short patterns will tend to be interesting if they have a high support, whereas long patterns can still be very interesting even if their support is relatively low. However, a large number of non-closed (i.e., redundant) patterns can still not be filtered out by simply applying the length-decreasing support constraint. As a result, a more desirable pattern discovery task could be mining closed patterns under the length-decreasing support constraint.

In this paper we study how to push deeply the length-decreasing support constraint into closed itemset mining, which is a particularly challenging problem due to the fact that the downward-closure property cannot be used to prune the search space. Therefore, we have proposed several pruning methods and optimization techniques to enhance the closed itemset mining algorithm, and developed an efficient algorithm, BAMBOO. Extensive performance study based on various length-decreasing support constraints and datasets with different characteristics has shown that BAMBOO not only generates more concise result set, but also runs orders of magnitude faster than several efficient pattern discovery algorithms, including CLOSET+, CFP-tree and LPMiner. In addition, BAMBOO also shows very good scalability in terms of the database size.

Keywords: Pattern discovery, frequent closed itemset, length-decreasing support constraint

1 Introduction

Since the introduction of association rule mining [1], frequent itemset mining has become a fundamental problem in data mining research and has been extensively studied [2, 19, 26, 9, 4, 11]. Various efficient frequent itemset mining algorithms have been developed, such as Apriori [2], FP-growth [12], H-mine [23], OP [16], and Inverted Matrix [8]. The main problem with these algorithms is that they may generate an exponentially large number of itemsets when the support is low. To overcome this problem, two classes of techniques/problem-formulations have been developed. The first focuses on mining maximal/closed itemsets and many recent studies have illustrated that it can lead to more compact result sets and better efficiency in finding frequent long itemsets from large datasets [3, 20, 24, 29, 6, 30, 27]. The second class attempts to reduce the number of potentially uninteresting itemsets by incorporating various anti-monotone, monotone, or convertible constraints within the constant-support-based frequent pattern mining framework [10, 17, 21, 22, 5].

A limitation of the above approaches is that they use a constant support value, irrespective of the length of the discovered patterns. In general, patterns that contain only a few items will tend to be interesting if they have a high support, whereas long patterns can still be interesting even if their support is relatively low. Instead of pushing more constraints into the constant-support based pattern mining, the LPMiner algorithm [25] uses the length-decreasing support constraint to prune some uninteresting patterns. Although LPMiner can sift out a lot of short infrequent itemsets, it still encounters difficulty in mining long itemsets, this is because a long frequent itemset implies a lot of short itemsets which may satisfy the length-decreasing support constraint. Consider the following length-decreasing support constraint: *minimum support equals 50 for length no greater than 5, and 20 for length greater than 5.* An itemset with a support no less than 20 and a length 10 means all its subsets with a length greater than 5 will be frequent, while an itemset with a support 60 and a length 8 means all of its subsets will be frequent. As a result, mining closed itemsets with length-decreasing support constraint is a more desirable task.

In this paper we mainly explore how to push deeply the length-decreasing support constraint into closed itemset mining. As stated in [25], developing such an algorithm is particularly challenging because the downward-closure property derived from the constant-

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support constraint cannot be used and for this reason LPMiner introduced the *smallest valid extension (SVE)* property to prune the search space. However, although the *SVE*-based pruning methods are very effective in enhancing the performance, they fail to take into account the specific characteristics of the transactions and items of the pattern-specific projected database. In this paper, we propose several pruning methods and optimization techniques which are derived directly from the length-decreasing support constraint and develop an efficient algorithm called BAMBOO[1] that finds all the closed itemsets satisfying the length-decreasing support constraint. BAMBOO incorporates two novel pruning methods called *invalid item pruning* and *unpromising prefix pruning* that are sensitive to the structure of the projected databases and quickly eliminate unpromising portions of the search space. Our experimental study reveals that BAMBOO can find over an order of magnitude fewer valid itemsets while it can be orders of magnitude faster than both the recently developed closed itemset mining algorithms and LPMiner.

The rest of the paper is organized as follows. Section 2 briefly describes the problem and some related work. Section 3 introduces the BAMBOO algorithm and provides a detailed description of the newly proposed pruning methods and optimization techniques. Section 4 presents a thorough experimental evaluation and compares BAMBOO's performance against that achieved by other algorithms. Finally, Section 5 provides some concluding remarks.

2 Problem Statement and Related Work

2.1 The Problem

A *transaction database* TDB is a set of transactions, where each transaction, denoted as a tuple $\langle tid, X\rangle$, contains a set of items (i.e., X) and is associated with a unique transaction identity *tid*. We use $|TDB|$ to denote the number of transactions in TDB. Let $I = \{i_1, i_2, \ldots, i_n\}$ be the complete set of distinct items appearing in TDB. An *itemset* Y is a non-empty subset of I and is called an *l-itemset* if it contains l items (i.e., $|Y| = l$). An itemset $\{x_1, \ldots, x_l\}$ is also denoted as $x_1 \cdots x_l$. A transaction $\langle tid, X\rangle$ is said to *contain* itemset Y if $Y \subseteq X$. The number of transactions in TDB containing itemset Y is called the *support* of itemset Y, denoted as $sup(Y)$.

DEFINITION 2.1. *(Closed itemset)* An itemset Y is a **closed itemset** if there exists no proper superset $Y' \supset Y$ such that $sup(Y') = sup(Y)$. □

DEFINITION 2.2. *(Length-decreasing support constraint)* A function $f(x)$ with respect to a transaction database TDB is called a **length-decreasing support constraint w.r.t. *TDB***, if it satisfies $|TDB| \geq f(x) \geq f(x+1) \geq 1$ for any positive integer x. An itemset Y, is *frequent* w.r.t. length-decreasing support constraint $f(x)$, if $sup(Y) \geq f(|Y|)$.[2] □

DEFINITION 2.3. *(Frequent closed itemset with Length-decreasing support constraint)* Given a transaction database TDB and its length-decreasing support constraint $f(x)$, if an itemset Y is a closed itemset and also frequent w.r.t. $f(x)$, Y is called a **frequent closed itemset w.r.t.** $f(x)$. □

Example. The first two columns in Table 1 show the transaction database TDB in our running example. We sort the list of items in support descending order and get the sorted item list which is called f_list. In this example f_list = ⟨f:4, c:4, a:3, b:3, m:3, p:3, i:1⟩. The list of items in each transaction are sorted according to f_list and shown in the third column of Table 1. Itemset fc is a 2-itemset with support 3, but it is not closed, because it has a superset fcam whose support is also 3. fcam is a frequent closed 4-itemset. Assume the length-decreasing support constraint f(x) is: $f(x) = 4$ *(for* $x \leq 3$*),* $f(x) = 3$ *(for* $4 \leq x \leq 5$*), and* $f(x) = 2$ *(for* $x \geq 6$*), it is easy to figure out that {f:4, c:4, fcam:3} is the complete set of frequent closed itemsets w.r.t f(x).*

Tid	Set of items	ordered item list
001	a, c, f, m, p	f, c, a, m, p
002	a, c, f, i, m, p	f, c, a, m, p, i
003	a, b, c, f, m	f, c, a, b, m
004	b, f	f, b
005	b, c, p	c, b, p

Table 1: A transaction database TDB.

2.2 Related Work

Mining frequent closed itemsets was first proposed in [20], which also proposed the A-close algorithm. Since then, several efficient closed itemset mining algorithms have been developed, including CLOSET [24], CHARM [30], CARPENTER [18], CLOSET+ [27], and CFP-tree [15]. These algorithms adopt similar search space pruning methods, and are different from each other in using different search strategies (e.g., depth-first search vs.

[1] The name "BAMBOO" was motivated by the fact that the shape of the bamboo plant schematically matches the nature of "length-decreasing" and "closed itemsets".

[2] From this definition we can see the constant support threshold is a special case of the length-decreasing support constraint which is given as a non-increasing function of the itemset length.

breadth-first search, bottom-up traversal vs. top-down traversal), different dataset representation (e.g., vertical format vs. horizontal format), data compression techniques (e.g., *Diffset* vs. FP-tree), database projection methods (e.g., physical projection vs. pseudo projection), and different closure checking schemes. Some winning techniques used in these algorithms will be employed in designing the BAMBOO algorithm.

An FP-tree based algorithm called TFP was introduced in [13] for finding the top-k frequent closed itemsets of length no less than a given value. TFP allows users to input the number of patterns to be discovered rather than the less intuitive minimum support and it is effective for finding long frequent itemsets. However, one drawback with TFP is that any patterns shorter than the given minimum length are never discovered, which might not be appropriate for some applications.

LPMiner was the first algorithm for finding all itemsets that satisfy a length-decreasing support constraint. In [25], the *Smallest Valid Extension (SVE)* property was proposed and used to design several pruning methods. However, LPMiner has a number of limitations. First, its result set may contain a large number of redundant (i.e., non-closed) itemsets; second, the *SVE*-based pruning methods are not as effective in pruning invalid items and unpromising prefix itemsets; third, they overlap with each other in functionality—further limiting their effectiveness.

There are several itemset discovery algorithms that use multiple support constraints. In [14], an algorithm was presented, in which each item has its own minimum item support (or MIS). The minimum support of an itemset is the lowest MIS among those items in the itemset. By sorting items in ascending order of their MIS values, the minimum support of the itemset never decreases as the length of itemset grows, making the support of itemsets downward closed. Thus an Apriori-based algorithm can be applied. In [28], an Apriori-based algorithm for finding frequent itemsets was proposed, which allows a set of more general support constraints. It is possible to represent a length-decreasing support constraint by using the formulation in [28]. However, the "*pushed*" minimum support of each itemset is forced to be equal to the support value corresponding to the longest itemset. Thus, it cannot prune the search space effectively. An algorithm in [7] adopts a different approach in that it does not use any support constraint. Instead, it searches for similar itemsets using probabilistic algorithms, which do not guarantee all frequent itemsets can be found.

3 BAMBOO Algorithm

In this section, we will develop the BAMBOO algorithm step by step. First, we will briefly introduce the LPCLOSET algorithm, a naïve algorithm which can mine closed itemsets with length-decreasing support constraint and forms the basis of the BAMBOO algorithm. It is simply derived from the recently developed frequent closed itemset mining algorithm, CLOSET+ (For more detail, we refer the readers to [27]). Then we will mainly focus on the search space pruning methods and several other optimization techniques newly proposed here, and also present the whole BAMBOO algorithm later.

3.1 LPCLOSET: A Naïve Solution

3.1.1 FP-tree Structure

To design an efficient frequent itemset mining algorithm, one should first choose a good data structure to represent the original database. Because LPCLOSET is a tailored version of the CLOSET+ algorithm, it also adopts the FP-tree structure, which is a prefix tree representation of the list of frequent items in the transactions, and shown to be an efficient representation of a transaction database by several studies [12, 24, 16, 27, 15]. Because the infrequent items are not used to build the FP-tree and a set of transactions sharing the same subset of items may share common prefix paths from the root of an FP-tree, FP-tree often has a high compression ratio in representing the original dataset, especially for dense datasets. Here we use our running example to illustrate the construction of the FP-tree structure.

Example. The FP_tree of our running example is built as follows: Scan the database once to count the support for each item and sort them in support-descending order to get the f_list (see our running example in section 2.1). The items whose support is lower than the minimum support for the longest itemset according to length-decreasing support constraint f(x) are infrequent and not used in building FP-tree. To insert a transaction into the FP-tree, infrequent items(e.g., item i) are removed and the remaining items in the transaction are sorted according to the item ordering in f_list. At each tree node, we record its item ID, count, and sum of transaction IDs (i.e., tids). For each FP-tree, there is a header table associated with it and there is an entry for each item in the header table. Fig. 1 shows the global FP-tree and its corresponding header table. The second and third columns in the header table represent the global count and sum of transaction IDs respectively, while the forth column is the side-link pointer which *links the nodes with the same item ID.*

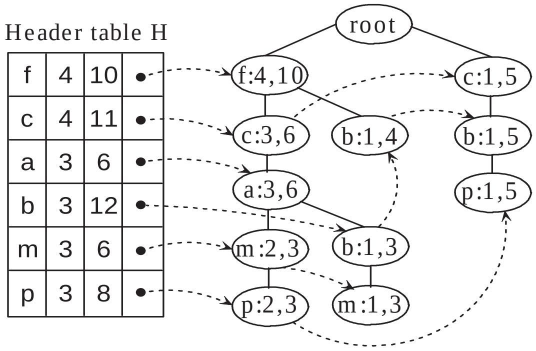
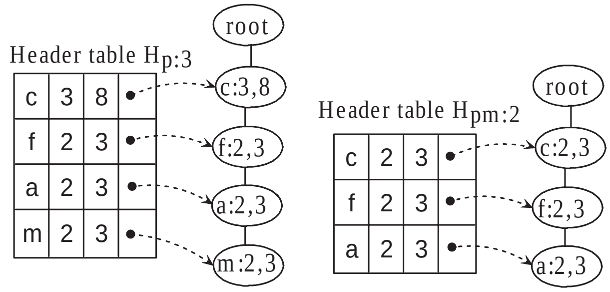


Figure 1: The FP-tree of our running example.



(a) Projected FP-tree with prefix p:3

(b) Projected FP-tree with prefix pm:2

Figure 2: Conditional FP-trees for prefix p:3 and pm:2.

3.1.2 Bottom-up Divide-and-Conquer

LPCLOSET adopts the popularly used divide-and-conquer paradigm in mining frequent itemsets. Specifically, it uses the *bottom-up divide-and-conquer* method, which follows the inverse *f_list* order (infrequent item *i* is removed): (1) first mine the patterns containing item *p*, (2) mine the patterns containing *m* but no *p*, (3) mine the patterns containing *b* but no *p* nor *m*, ..., and finally mine the patterns containing only *f*. Upon mining the patterns containing item *p* (i.e., *p* is the current prefix itemset), we will follow its side-link pointer recorded in the global header table and locate the *conditional database* with prefix *p* (denoted as $TDB|_{p:3}$), which contains two amalgamative transactions: $\langle$*fcam:2*$\rangle$ and $\langle$*cb:1*$\rangle$. The projected FP-tree for prefix *p*:3 is built from $TDB|_{p:3}$ as Fig. 2(a) shows (infrequent item *b* has been removed). The *bottom-up divide-and-conquer* method is applied in a recursive way. For example, after building the conditional FP-tree for prefix *p*:3, we will first mine patterns containing *pm*:2, then mine patterns containing *pa*:2, and so on. The conditional FP-tree for prefix *pm*:2 is shown in Fig. 2(b).

3.1.3 Search Space Pruning

Since we are only interested in closed itemsets, we should design some pruning methods to stop mining patterns with unpromising prefix itemsets as soon as possible. Previous studies proposed several methods, here we adopt two techniques for LPCLOSET, which have been popularly used in several closed/maximal itemset mining algorithms [3, 24, 6, 30, 27, 15].

The first pruning technique is *item merging*. For a prefix itemset P, the complete set of its locally frequent items that have the same support as P can be merged with P to form a new prefix, and these items can be safely removed from the list of locally frequent items of the new prefix. For example, as shown in Fig. 2(a), the set of locally frequent items w.r.t. prefix *p*:3 is $\{c$:3, f:2 a:2, m:2$\}$, item c can be merged with the prefix *p*:3 to form a new prefix *pc*:3, and the set of locally frequent items becomes $\{f$:2, a:2, m:2$\}$.

The second pruning technique is *sub-itemset pruning*. Let X be the frequent itemset currently under consideration. If X is a proper subset of an already found frequent closed itemset Y and $sup(X) = sup(Y)$, then there is no hope to generate frequent closed itemsets from X and thus X can be pruned. For example, when we want to mine patterns with prefix *a*:3, another frequent closed itemset *fcam*:3 has been mined under the *bottom-up divide-and-conquer* paradigm and is a proper superset of *a* with the same support, we can safely stop mining patterns with prefix *a*:3.

3.1.4 Closure Checking Scheme

The above search space pruning methods can accelerate the mining process by removing some unpromising prefix itemsets, but they cannot eliminate all the non-closed itemsets. We need to do *subset checking* for each mined frequent itemset in order to assure that it is really a closed itemset [27]. Like CLOSET+, we maintain the set of already mined closed itemsets in a compact result tree structure. Fig. 3 shows an example which stores totally three closed itemsets: *fcamp*:2, *cp*:3, and *fcam*:3. Upon getting a new frequent itemset, we will check against the set of closed itemsets stored in the result tree to see if there exists such a closed itemset that has the same support and is a proper superset of the current itemset. If that is the case, the current candidate itemset is not closed and will not be inserted into the result tree. Instead of using the two-level hash indexing as in CLOSET+, here we use the *sum of the transaction IDs* as the hash key (similar to [30])and the hash value as the index into the result tree in order to reduce the search space for pattern closure checking. For example, at the status of Fig. 3, upon getting a new prefix a:3 whose sum of the transaction IDs is 6, it is easy to find that closed itemset $fcam$:3 has the same support as a:3 and can absorb a:3. Thus a:3 is non-closed and we can safely stop mining patterns with prefix a:3.

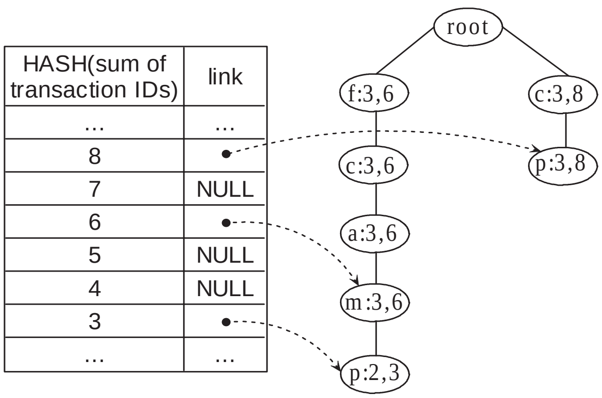


Figure 3: Hash-indexed result tree structure.

3.1.5 Simply Integrating the Constraint

Up to now, LPCLOSET can mine the complete set of closed itemsets. To get the set of closed itemsets which satisfy the length-decreasing support constraint, it is rather straightforward: when we find a closed itemset P, we simply check whether $sup(P) \geq f(|P|)$, if so, we will output it as a valid itemset and also store it in the result tree. The problem becomes whether we should still hold an infrequent closed itemset in the result tree in order to check a new itemset's closure?

LEMMA 3.1. *(Result-tree pruning) If the current candidate prefix itemset, P, cannot pass the checking of the length-decreasing support constraint, there is no need to keep it in the result tree in order to check the closure of the later found itemsets.*

Proof. The correctness of this lemma is evident: Assume a later found prefix itemset $P^{'}$ can be subsumed by itemset P (that is, $(P^{'} \subset P)$ and $(sup(P^{'}) = sup(P))$). Because $sup(P^{'}) = sup(P)$, $sup(P) < f(|P|)$, and $f(|P|) \leq f(|P^{'}|)$ hold, $sup(P^{'}) < f(|P^{'}|)$ must also hold. This means if we do not maintain P in the result tree, we may not be able to correctly check the closure of a later mined itemset $P^{'}$, but all such kind of itemsets as $P^{'}$ must be invalid, that is they cannot satisfy the length-decreasing support constraint. □

Because checking whether a prefix itemset can pass the support constraint is much cheaper than checking a pattern's closure, based on Lemma 3.1, when we get a new prefix itemset, we will first check if it can satisfy the support constraint, if not, we do not need to check if it is closed and it will not be inserted into the result tree. For example, $fcamp$:2 and cp:3 will not be inserted into the result tree and should not appear in Fig. 3. ALGORITHM 1 shows the whole LPCLOSET algorithm, which calls subroutine lpcloset(pi, cdb)(i.e., SUBROUTINE 1). It uses the *item-merging* method (line 03) and *sub-itemset pruning* method (line 06) to prune the search space. If an itemset satisfies the support constraint (line 05) and passes the closure checking (line 08), it will be inserted into the result-tree and output as a valid pattern(line 09). lpcloset() adopts the *bottom-up divide-and-conquer* paradigm to grow the current prefix pi and recursively calls itself(lines 13-19).

ALGORITHM 1: **LPCLOSET**(*TDB, f(x)*)

INPUT: (1) *TDB*: *a transaction database*, and (2) *f(x)*: *a length-decreasing support constraint function.*
OUTPUT: (1) *SCI*: *the complete set of closed itemsets that satisfy the length-decreasing support constraint.*

```
BEGIN
01. SCI ← ∅;  result_tree ← NULL;
02. call lpcloset(∅, TDB);
END
```

SUBROUTINE 1 : **lpcloset**(*pi, cdb*)

INPUT: (1) *pi*: *a prefix itemset*, and (2) *cdb*: *the conditional database w.r.t. prefix pi.*

```
BEGIN
03. item_merging(pi, cdb);
04. if(pi ≠ ∅)
05.    if(sup(pi) ≥ f(|pi|))
06.       if(sub-itemset_pruning(pi, result_tree))
07.          return;
08.       else
09.          insert(result_tree, pi);  SCI ← SCI ∪ {pi};
10.       end if
11.    end if
12. end if
13. I ← set of local items w.r.t. pi
14. fptree ← FP-tree_construction(I, cdb);
15. for all i∈I do
16.    pi' ← pi ∪ {i};
17.    cdb' ← build_cond_database(pi', fptree)
18.    call lpcloset(pi', cdb');
19. end for
END
```

3.2 Deeply Pruning

The above LPCLOSET which can be seen as a tailored variant of the CLOSET+ algorithm can mine the closed itemsets with length-decreasing support constraint, but it does not make full use of the length-decreasing support constraint to enhance the performance. In LPMiner, three efficient pruning methods, *Transaction pruning*, *Node pruning*, and *Path pruning*, have been proposed, which are all based on the *Smallest Valid Extension* property (or *SVE* for short) [25].

PROPERTY 3.1. *(Smallest valid extension) Given an itemset P such that* $sup(P) < f(|P|)$, *then* $f^{-1}(sup(P)) = min(l | f(l) \leq sup(P))$ *is the minimum length that a super-itemset of P must have before it can potentially satisfy the length-decreasing support constraint.* □

From the *SVE* property, the three pruning methods used in [25] can be easily derived. For example, the *transaction pruning* method can be described as: *any transaction t in prefix P's conditional database can be pruned if* $(|P| + |t|) < f^{-1}(sup(P))$. Although the pruning methods adopted by LPMiner are very effective, they do not fully explore the support constraint to deeply prune the search space. The deficiency is two-fold: on one hand, they are all based on the *SVE* property, their functionality is overlapped to some extent; on the other hand, they prune the space in a too coarse granularity, they cannot keep the invalid items or unpromising prefix itemsets from mining. Therefor, instead of simply adopting the previously developed pruning methods, we have designed two new pruning methods in order to deeply push the length-decreasing support constraint into the closed itemset mining.

3.2.1 Invalid Item Pruning

Most frequent itemset mining algorithms use constant support constraint to control the inherently exponential pattern discovery complexity by pruning the infrequent items based on the well-known downward closure property. However, with the length-decreasing support constraint, there is no straightforward way to determine whether an item is frequent or not: an item which is infrequent w.r.t. a short prefix itemset may later become frequent w.r.t. a long prefix itemset. Thus we need to find a way to define and prune some unpromising items from which no closed itemset satisfying the length-decreasing support constraint can be generated.

DEFINITION 3.1. *(Invalid item)* Let max_l be the maximal transaction length in conditional database $TDB|_P$ w.r.t. a particular prefix itemset P. For any item x which appears in $TDB|_P$, we maintain a total number of max_l counts, denoted as $COUNT^x[1..max\_l]$, where $COUNT^x[i] (1 \leq i \leq max\_l)$ records the total number of occurrences of item x in transactions no shorter than i. Item x is called an *invalid item* w.r.t. $TDB|_P$ if $\forall i$ $(1 \leq i \leq max\_l), COUNT^x[i] < f(i + |P|)$. □

LEMMA 3.2. *(Invalid item pruning) Given a particular prefix itemset P, and its conditional database* $TDB|_P$. *There is no hope to grow P with any invalid item w.r.t.* $TDB|_P$ *to get closed itemsets satisfying the length-decreasing support constraint.*

Proof. We will prove it by contradiction. Assume invalid item x can be used to grow P to get a closed itemset P' which satisfies the length-decreasing constraint $f(x)$, that is, $P' \supseteq (\{x\} \cup P)$ and $sup(P') \geq f(|P'|)$. This means $COUNT^x[|P'| - |P|] \geq f(|P'|)$, which contradicts with the definition of an *invalid item.* □

The *invalid item pruning* method can be applied to both the original database and conditional database w.r.t. a certain prefix. Here we use our running example to show its usefulness. From Table 1, we know $COUNT^b[i] = 3$ (for $1 \leq i \leq 2$), $COUNT^b[i] = 2$ (for i=3), $COUNT^b[i] = 1$ (for $4 \leq i \leq 5$), and $COUNT^b[i] = 0$ (for i=6), which means $\forall i, COUNT^b[i] < f(i)$, as a result, item b is invalid and can be pruned from the global FP-tree shown in Fig. 1. Similarly, item p is also invalid and can be pruned from the global FP-tree.

3.2.2 Unpromising Prefix Itemset Pruning

BAMBOO is based on LPCLOSET, which uses the pattern growth method [12] to mine frequent closed itemsets. At any time, there is always a prefix itemset, from which longer frequent closed itemsets can be grown. However, in many cases no closed itemset that satisfies the length-decreasing support constraint can be generated by growing some unpromising prefix itemsets, we should detect such kind of prefix itemsets as soon as possible and avoid mining patterns with these unpromising prefix itemsets.

DEFINITION 3.2. *(Unpromising prefix)* Let max_l be the maximal transaction length in conditional database $TDB|_P$ w.r.t. a particular prefix itemset P. We maintain a total number of max_l counts for P, denoted as $COUNT^P[1..max\_l]$, where $COUNT^P[i] (1 \leq i \leq max\_l)$ records the total number of transactions in $TDB|_P$ with a length no shorter than i. Prefix itemset P is called an *unpromising prefix itemset* if $\forall i$ $(1 \leq i \leq max\_l), COUNT^P[i] < f(i + |P|)$. □

LEMMA 3.3. *(Unpromising prefix itemset pruning) There is no hope to grow an unpromising prefix itemset P to find closed itemsets that can satisfy the length-decreasing support constraint, and thus P can be safely pruned.*

Proof. This Lemma can also be proven by contradiction. Assume prefix P can be used as a start point to get a closed itemset $P^{'}$ which satisfies the length-decreasing constraint $f(x)$, that is, $P^{'} \supset P$ and $sup(P^{'}) \geq f(|P^{'}|)$. This means $COUNT^{P}[|P^{'}| - |P|] \geq f(|P^{'}|)$, which contradicts with the definition of an *unpromising prefix itemset*. □

Let us look at an example. Assume prefix itemset X_1=p:3, by following the side-link pointer of item p in Fig. 1 we can easily figure out that $TDB|_{p:3}$ contains two amalgamative transactions: $\langle fcam{:}2\rangle$ and $\langle cb{:}1\rangle$. We can get that, $COUNT^{X_1}[i] = 3$ (for $1 \leq i \leq 2$) and $COUNT^{X_1}[i] = 2$ (for $3 \leq i \leq 4$). As a result, $\forall i$ $(1 \leq i \leq 4), COUNT^{X_1}[i] < f(i + |X_1|)$. Prefix itemset X_1=p:3 is an unpromising prefix itemset and can be pruned.

3.3 Further Optimizations

Most pruning methods can prune the search space effectively while in the meantime they themselves may also lead to non-negligible overheads. In the following, we will explore the *SVE* property and *binning* technique to minimize the potential overheads incurred by these methods.

3.3.1 SVE-based Enhancement

Here we first consider to use the *SVE* property to enhance the *unpromising prefix itemset pruning* method. Because we record some statistic information in the header table for each locally frequent item w.r.t. a certain prefix, prior to checking whether a prefix is promising or not, we already know the support of a new prefix. For example, from the global header table shown in Fig. 1, we know that the support of prefix p is 3. According to the *SVE* property, we know that some conditional transactions may contribute nothing to the set of closed itemsets which satisfy the length-decreasing support constraint, as a result, they will not be used to judge whether a prefix is promising or not. In our running example, let p:3 be the current prefix itemset under consideration, we can compute its *SVE* as: $f^{-1}(sup(p))$= $f^{-1}(3)$=4, which means a conditional transaction with a length shorter than $f^{-1}(sup(p)) - |p|$=3 can be ignored when we compute $COUNT^{p:3}[1..4]$. As a result, one of the prefix p:3's amalgamative transactions, $\langle cb{:}1\rangle$ can be ignored. This technique will help a lot when there are many short conditional transactions w.r.t. a prefix itemset.

3.3.2 Binning-based Enhancement

One of the main overhead related to the *invalid item pruning* method is caused by memory manipulation. Remember that we need to maintain a total number of *max_l* counts (i.e., $COUNT^{x}[1..max\_l]$) for each item x, where *max_l* is the maximal transaction length in the corresponding (conditional) database. Allocating, freeing or resetting a non-trivial memory is costly, thus we need to reduce the memory usage as possible as we can. One way to achieve this is to employ the binning technique to improve the memory usage: instead of keeping a count for each length in the range $[1..max\_l]$, we can keep a subset of these counts, $COUNT^{x}[1..m]$, corresponding to length l_1, l_2, ..., and l_m, where $1 = l_1 < l_2 < ... < l_m < max\_l$. Among which, we use $COUNT^{x}[l_i]$ to record the number of transactions no shorter than l_i in which item x appears. Then we can use the following lemma to judge whether an item is invalid or not.

LEMMA 3.4. *(Relaxed invalid item) Given a particular prefix itemset P, and its conditional database* $TDB|_P$*. Assume* x *is a local item and* max_l *is the maximal transaction length in* $TDB|_P$*. Let* $COUNT^{x}[l_i]$ *be the number of transactions no shorter than* l_i *in which item* x *appears, where* $1 \leq i \leq m$ *and* $1 = l_1 < l_2 < ... < l_m < max\_l$ *. If* $COUNT^{x}[l_m] < f(max\_l)$ *and* $COUNT^{x}[l_i] < f(l_{i+1})$ *(for* $1 \leq i < m$*), item* x *is invalid w.r.t. prefix P.*

Proof. $COUNT^{x}[l_m] < f(max\_l)$ means no itemset $P^{'}$, where $|P^{'}| \geq l_m$ and $P^{'} \supseteq (\{x\} \cup P)$, can satisfy the length-decreasing support constraint, while $COUNT^{x}[l_i] < f(l_{i+1})$(for $1 \leq i < m$) means no itemset, $P^{'}$, where $l_i \leq |P^{'}| \leq l_{i+1}$ and $P^{'} \supseteq (\{x\} \cup P)$ can satisfy the length-decreasing support constraint. As a result no itemset with any length can satisfy the length-decreasing support constraint and in the meantime contain both item x and prefix P. □

The above technique can be implemented as follows: If we find item x appears in a transaction with a length L, where $l_i \leq L < l_{i+1}$, $COUNT^{x}[l_i]$ will be incremented by 1. When we check whether item x is invalid or not, we start from l_m to see if $COUNT^{x}[l_m] < f(max\_l)$ holds, if so, $COUNT^{x}[l_m]$ will be added to $COUNT^{x}[l_{m-1}]$, otherwise item x will be valid. In general, if $COUNT^{x}[l_i] < f(l_{i+1})$ holds, $COUNT^{x}[l_i]$ will be added to $COUNT^{x}[l_{i-1}]$ and then check whether $COUNT^{x}[l_{i-1}]$ violates the length-decreasing support constraint. If we finally find $COUNT^{x}[l_1] < f(l_2)$, we can safely say item x is invalid and can be pruned from further consideration. Here we show an example on how to detect invalid item b from the original database TDB. Assume we maintain three counts for item b, and $l_1 = 1$, $l_2 = 3$, $l_3 = 5$. After scanning the TDB once, we get $COUNT^{b}[l_1] = 1$, $COUNT^{b}[l_2] = 1$, and $COUNT^{b}[l_3] = 1$. Because $COUNT^{b}[l_3] < f(6)$

(here $max\_l = 6$), $COUNT^b[l_3]$ will be added to $COUNT^b[l_2]$. Similarly, $COUNT^b[l_2] < f(l_3)$ and $COUNT^b[l_2]$ will be added to $COUNT^b[l_1]$. Now $COUNT^b[l_1]$ becomes 3, which is less than $f(l_2)$. As a result, we know that item b is invalid and can be pruned.

ALGORITHM 2: **BAMBOO**(*TDB, f(x)*)

INPUT: (1) *TDB*: *a transaction database*, and (2) *f(x)*: *a length-decreasing support constraint function*.
OUTPUT: (1) *SCI*: *the complete set of closed itemsets that satisfy the length-decreasing support constraint*.
BEGIN
01. $SCI \leftarrow \emptyset$; *result_tree* $\leftarrow$ *NULL;*
02. *call* **bamboo**($\emptyset$, *TDB);*
END

SUBROUTINE 2 : **bamboo**(*pi, cdb*)

INPUT: (1) *pi*: *a prefix itemset*, and (2) *cdb*: *the conditional database w.r.t. prefix pi.*
BEGIN
03. $I \leftarrow$ *invalid_item_pruning(cdb,f(x));*
04. $S \leftarrow$ *item_merging(I);* $pi \leftarrow pi \cup S$; $I \leftarrow I - S$;
05. *if*($pi \neq \emptyset$)
06. *if(unpromising_prefix_pruning(pi, cdb, f(x)))*
07. *return;*
08. *end if*
09. *if*($sup(pi) \geq f(|pi|)$)
10. *if(sub-itemset_pruning(pi, result_tree))*
11. *return;*
12. *else*
13. *insert(result_tree, pi);* $SCI \leftarrow SCI \cup \{pi\}$;
14. *end if*
15. *end if*
16. *end if*
17. *if*($I \neq \emptyset$)
18. $cdb \leftarrow$ *transaction_pruning(pi,cdb);*
19. *fptree* $\leftarrow$ *FP-tree_construction(I, cdb);*
20. *for all* $i \in I$ *do*
21. $pi^{'} \leftarrow pi \cup \{i\}$;
22. $cdb^{'} \leftarrow$ *build_cond_database*($pi^{'}$, *fptree)*
23. *call* **bamboo**($pi^{'}$, $cdb^{'}$);
24. *end for*
25. *end if*
END

3.4 The Algorithm

By incorporating the above pruning methods and optimization techniques into LPCLOSET, we get the BAMBOO algorithm as shown in ALGORITHM 2.

Algorithm BAMBOO calls subroutine bamboo(*pi*, *cdb*). As shown in SUBROUTINE 2, given a certain prefix itemset *pi* and its corresponding conditional database *cdb*, bamboo(*pi*, *cdb*) first applies the *invalid item pruning method*, and the set of valid items is denoted as I (line 03). Next it uses the *item merging* technique to identify the set of items that has the same support as *pi*, and denoted as *S*, S will be merged with *pi* and removed from I (line 04). Then it uses the *unpromising prefix pruning*(line 06), *result-tree pruning*(line 09) and *sub-itemset pruning*(line 10) methods to prune a non-empty prefix (lines 05-16). If an itemset can satisfy the length-decreasing support constraint(line 09) and pass closure checking(line 12), it must be a closed itemset that satisfies the support constraint, and will be inserted into the result tree(line 13). Next the *transaction pruning* method is applied(line 18) and the conditional FP-tree is built(line 19), and bamboo() recursively calls itself to mine the closed itemsets with length-decreasing support constraint by growing prefix *pi* under the *bottom-up divide-and-conquer* paradigm (lines 17-25).

4 Empirical Results

In this section we present a comprehensive experimental evaluation of BAMBOO and compare its performance against that achieved by other algorithms. Our results show that (i) by incorporating length-decreasing support constraint into closed itemset mining, BAMBOO not only generates more compact result set, but also has much better performance than both the recently developed closed itemset mining algorithms and the LPMiner algorithm; (ii) the newly proposed search space pruning methods for BAMBOO are very effective in enhancing the performance; (iii) BAMBOO has very good scalability in terms of the number of transactions.

4.1 Test Environment and Dataset

To our knowledge, CLOSET+ [27] and CFP-tree [15] are two recently developed frequent closed itemset mining algorithms. We compared BAMBOO with these two algorithms on a 2.4GHz Intel Pentium PC with 1GB memory and Windows 2000 installed. All these three algorithms were implemented using Microsoft Visual C++. When we ran CLOSET+ and CFP-tree, the support threshold is chosen as the minimum support for the maximum itemset length under the corresponding length-decreasing support constraint. We also compared BAMBOO with LPMiner [25], which is a frequent itemset mining algorithm with length-decreasing support constraint. In our experiments, we used four real datasets and some synthetic datasets which have been popularly used in some previous studies [31, 30, 13, 27].

The characteristics of the datasets are shown in Table 2 (the last column shows the average and maximal transaction length).

Dataset	# Tuples	# Items	A.(M.) t. l.
connect	67557	150	43(43)
pumsb	49046	2114	74(74)
mushroom	8124	120	23(23)
gazelle	59601	498	2.5(267)
T10I4D100k	100k	1000	10(31)
T10I4Dx	200k-1000k	1000	10(31)

Table 2: Dataset Characteristics.

Real datasets: The four real datasets used in our experiments are *connect*, *pumsb*, *mushroom*, and *gazelle*, respectively. The *connect* dataset is a very dense dataset, which contains game state information, the *pumsb* dataset contains census data and is also a dense dataset, the *mushroom* dataset is a little (but not very) dense and contains characteristics of various species of mushrooms, while the *gazelle* dataset contains click-stream data from Gazelle.com and is a very sparse dataset with a few very long transactions.

Synthetic datasets: The synthetic datasets were generated from IBM dataset generator, and each contains totally 1000 distinct items with an average transaction length 10 and average frequent itemset length 4. The dataset *T10I4D100K* contains 100,000 transactions, while the dataset series *T10I4Dx* contain 200K to 1000K transactions, which were used to test the algorithm scalability.

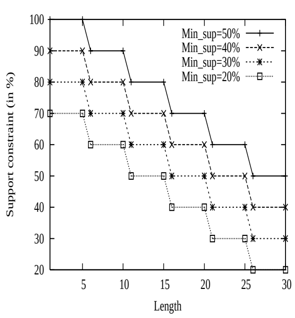


Fig. 4. Support constraint (*connect*).

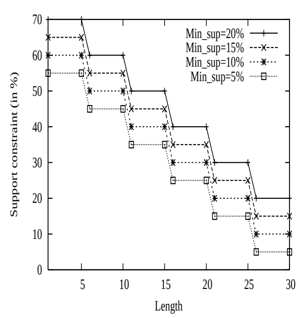


Fig. 5. Support constraint (*connect*).

Length-decreasing support constraints: Fig. 4-9 show various length-decreasing support constraints used in our tests for different datasets. Fig. 4 and Fig. 5 depict two sets of stair-style length-decreasing support constraints for dataset *connect*. Fig. 4 shows four length-decreasing support constraints with minimum support for itemsets longer than 25 set at 20%,

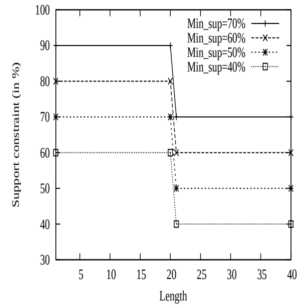


Fig. 6. Support constraint (*pumsb*).

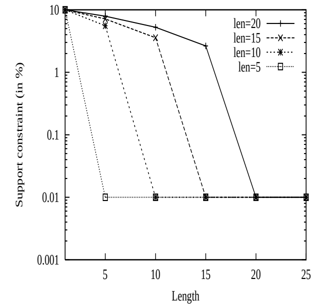


Fig. 7. Support constraint (*mushroom*).

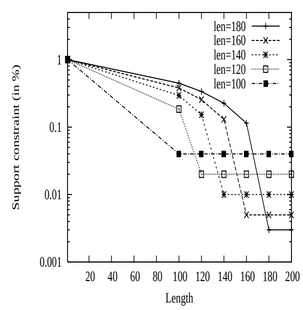


Fig. 8. Support constraint (*gazelle*).

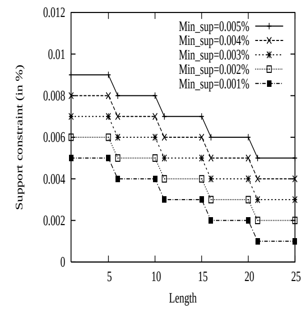


Fig. 9. Support constraint (*T10I4D100K*).

30%, 40%, and 50%, respectively, while Fig. 5 shows another four constraints with minimum support for itemsets longer than 25 set at 5%, 10%, 15%, and 20%, respectively. Fig. 6 shows another four special stair-style length-decreasing support constraints for *pumsb* dataset, with minimum support for itemsets longer than 20 set at 40%, 50%, 60%, and 70%, respectively. For dataset *mushroom*, we used a set of linear length-decreasing support constraint functions: The support threshold is set at 10% at length 1, and is linearly decreased to 0.01% at length 5, 10, 15, 20, respectively (Because the y-axis in Fig. 7 is in log-scale, the corresponding curves do not look like linear functions, although they are in fact linear. Similar situation happens in Fig. 8). For dataset *gazelle*, we chose five different support constraints: starting from support 1% for length 1, the support threshold will be linearly decreased to 0.04%, 0.02%, 0.01%, 0.005%, and 0.003% at length 100, 120, 140, 160 and 180, respectively. Fig. 9 shows a set of stair-style length-decreasing support constraints chosen for dataset *T10I4D100K*, with minimum support for length greater than 20 set at 0.001%, 0.002%, 0.003%, 0.004%, and 0.005%, respectively.

4.2 Performance Testing

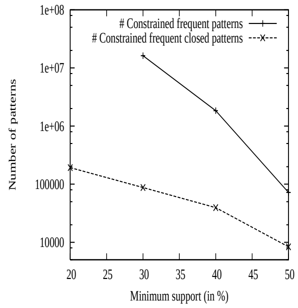


Fig. 10. frequent vs. closed(*connect*).

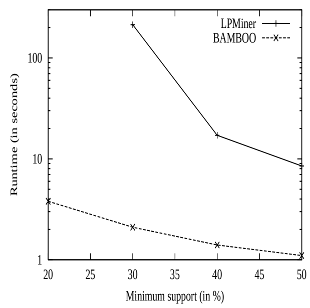


Fig. 11. Runtime comparison(*connect*).

Comparison with LPMiner We first compared BAMBOO with LPMiner in order to show that mining constrained closed itemsets is a more desirable task than mining constrained frequent itemsets. We used the datasets listed above and all showed the similar picture: BAMBOO can generate more concise result set while it is more efficient than LPMiner. Due to the limited space, here we only present the comparison results for the *connect* dataset. Fig. 10 compares the number of frequent itemsets with that of frequent closed itemsets, both satisfying the set of length-decreasing support constraints shown in Fig 4 (A minimum support 20% in Fig. 10 represents the length-decreasing support constraint whose minimum support is 20% for itemsets longer than 25). Fig. 11 shows the runtime of the two algorithms under the same set of length-decreasing support constraints. We can see BAMBOO generates orders of magnitude fewer patterns, and can be orders of magnitude faster than LPMiner. BAMBOO's higher performance stems from two aspects: on the one hand, it mines much smaller number of valid itemsets than LPMiner, on the other hand, it adopts more effective pruning methods, which can make full use of the length-decreasing support constraint to prune the search space quickly.

Comparison with CLOSET+ and CFP-tree We also compared BAMBOO with two recently developed frequent closed itemset mining algorithms, CFP-tree [15] and CLOSET+ [27]. In comparing these three algorithms, we recorded the total number of frequent closed itemsets and the number of closed itemsets which satisfy the corresponding length-decreasing support constraints shown in Fig 5-9 (We call it the number of constrained closed patterns in the corresponding figures). We first compared the three algorithms using the dataset *connect* and the support constraints shown in Fig. 5. From Fig. 13, we can see at high support threshold (e.g., 20%), BAMBOO can be several times faster than both CLOSET+ and CFP-tree, while at low support (e.g., 5%), BAMBOO can be several orders of magnitude faster than CLOSET+ and CFP-tree. Fig. 12 shows that the length-decreasing support constraint is very effective in shrinking the result set, while on the other hand, it also implicates that as long as we can find some effective search space pruning methods for BAMBOO, it will be potentially more efficient than the traditional closed pattern mining algorithms.

Fig. 14 and Fig. 15 show the comparison results for dataset *pumsb*: BAMBOO always generates much smaller number of patterns and runs much faster than CLOSET+ and CFP-tree under the length-decreasing support constraints shown in Fig. 6. For example, at support 50%, BAMBOO can be about 60 times faster than CLOSET+ and more than 100 times faster than CFP-tree, while the number of constrained closed patterns is about 30 times less than that of all frequent closed patterns.

Unlike *connect* and *pumsb*, the *mushroom* dataset is relatively small and not very dense, from which a not too large number of frequent closed patterns can be found even with a very low support threshold like 0.01%. Even for such a small dataset, BAMBOO can still use some kind of length-decreasing support constraints to reduce the number of valid closed patterns and improve the performance. From Fig. 16 and Fig. 17, we know under the support constraints shown in Fig. 7, BAMBOO can be several times faster than CLOSET+ and CFP-tree, while its result set can be several times smaller.

Gazelle is a very sparse dataset and can generate a few very long frequent closed itemsets with low support threshold. Most of the current closed itemset mining algorithms run too slow when the support threshold is lower than a certain threshold, this is because on the one hand mining long patterns itself is very challenging, on the other hand, a large number of short frequent closed patterns will be generated. Fig. 18 and Fig. 19 show very interesting and also amazing results: under the linearly length-decreasing support constraint functions shown in Fig. 8, BAMBOO can sift out most of the closed patterns and only a few very long itemsets with low support and a few short itemsets with high support are left. For example, if the support threshold is linearly decreased from 1% at length 1 to 0.003% at length 180, BAMBOO only finds 79 valid itemsets and the longest itemset has a length 184. Fig. 18 and Fig. 19 show that BAMBOO generates orders of magnitude fewer patterns and runs several orders of magnitude faster than CLOSET+ and CFP-tree.

T10I4D100K is also a sparse dataset, from which a lot of short frequent closed itemsets can be mined for low support thresholds. Fig. 20 and Fig. 21 demonstrate the results for this dataset under the set of stair-style support constraints shown in Fig. 9.

BAMBOO can be several times faster than CLOSET+, while can be several orders of magnitude faster than CFP-tree algorithm. In the meantime, the number of closed itemsets satisfying the length-decreasing support constraints is several times smaller than that of all frequent closed itemsets.

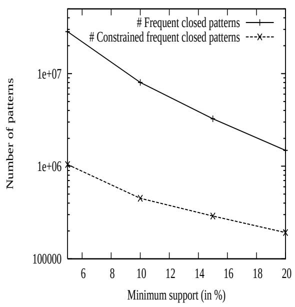


Fig. 12. constrained vs. non-constrained(*connect*).

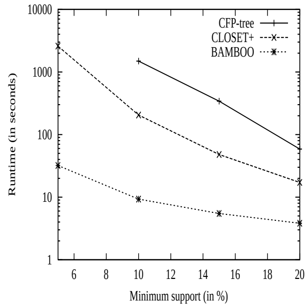


Fig. 13. Runtime comparison(*connect*).

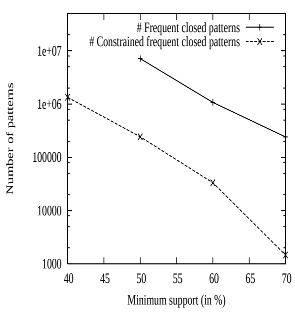


Fig. 14. constrained vs. non-constrained(*pumsb*).

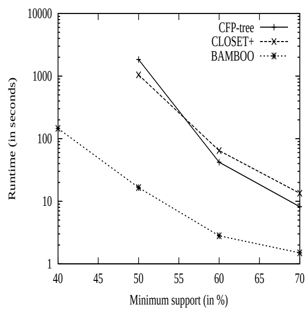


Fig. 15. Runtime comparison(*pumsb*).

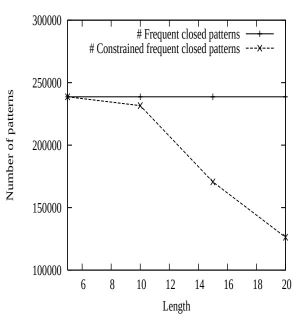


Fig. 16. constrained vs. non-constrained (*mushroom*).

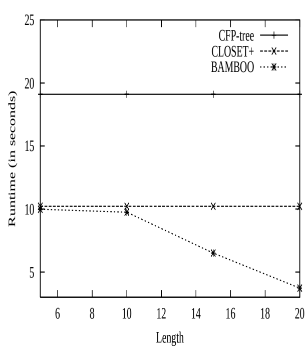


Fig. 17. Runtime comparison(*mushroom*).

Effectiveness of the pruning methods The above comparison results have validated the effectiveness of

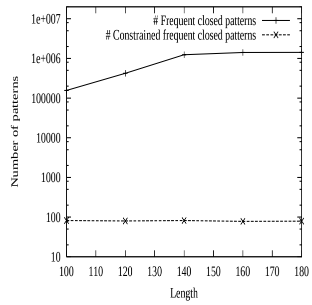


Fig. 18. constrained vs. non-constrained (*gazelle*).

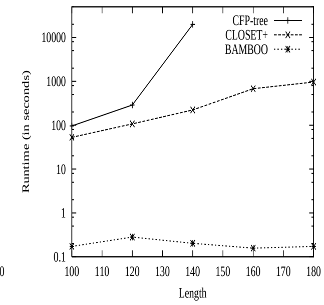


Fig. 19. Runtime comparison(*gazelle*).

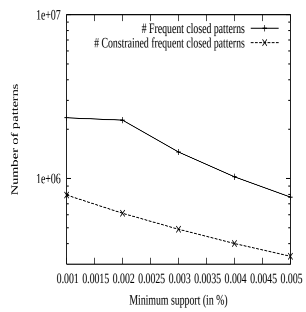


Fig. 20. constrained vs. non-constrained (*T10I4D100K*).

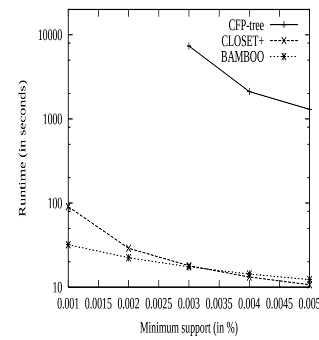


Fig. 21. Runtime comparison(*T10I4D100K*).

the length-decreasing support constraints in compressing the result set: BAMBOO can generate orders of magnitude fewer patterns than the traditional all frequent closed pattern mining algorithms. But this does not mean any pattern discovery algorithms with length-decreasing support constraint will have much better performance than a frequent closed pattern mining algorithm. The efficiency of such kind of constrained algorithms mainly depends on whether we can propose some effective search space pruning methods. The above performance comparison results have to some extent already demonstrated the effectiveness of the pruning methods adopted by BAMBOO. Here we will use the *T10I4D100K* dataset to isolate the effectiveness of each pruning method used in BAMBOO. Fig. 22 shows the results under the support constraints in Fig. 9. The Legend "*no-pruning*" corresponds to the LPCLOSET algorithm, which does not apply any of the following three pruning methods, *unpromising prefix itemset pruning*, *invalid item pruning*[3], and *transaction prun-*

[3] Here the *unpromising prefix itemset pruning* is the *SVE*-enhanced version, while the *invalid item pruning* has been optimized using the *binning* technique.

ing. We can see that compared to the LPCLOSET algorithm, each pruning method is very effective in boosting the performance and when we incorporate all the three pruning methods into BAMBOO, it achieves the best performance. In addition, in comparison to one of the best SVE-based pruning methods, *transaction pruning*, the pruning methods proposed in this paper, *unpromising prefix itemset pruning* and *invalid item pruning*, are more effective in enhancing the algorithm performance.

Scalability test We also tested BAMBOO's scalability in terms of the base size by using the synthetic dataset series *T10I4Dx*. Here we varied the base size from 200K tuples to 1000K tuples and ran BAMBOO under the five different length-decreasing support constraints shown in Fig. 9. In presenting the results, we use a legend "S=0.001%" to represent the length-decreasing support constraint whose minimum support is 0.001% for length greater than 20. From Fig. 23, we know that BAMBOO has very good linear scalability against the number of transactions in the dataset.

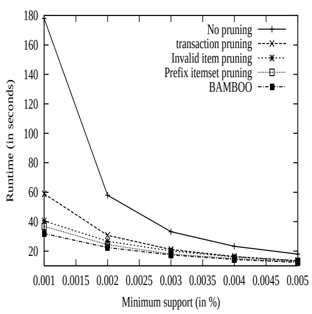


Fig. 22. Comparison of the pruning methods (*T10I4D100K*).

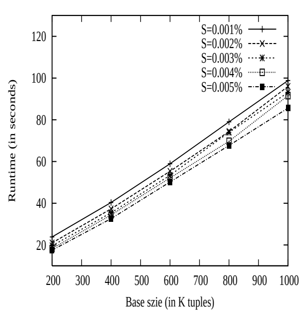


Fig. 23. Scalability test against base size(*T10I4Dx*).

5 Conclusions

Many previous studies have elaborated that mining frequent closed/maximal itemsets in large databases or mining frequent itemsets with length-decreasing support constraints can lead to more compact result set and possibly better performance. However, as our empirical study has showed these two kinds of algorithms still generate too many patterns when the support is low or the patterns become long. In this paper we developed BAMBOO, the first algorithm which can push deeply the length-decreasing support constraint into the traditional closed itemset mining in order to generate more concise and meaningful result set and gain better efficiency. Under this more general framework the downward-closure property no longer holds, we cannot use it to prune search space. Instead, two search space pruning methods, *unpromising prefix itemset pruning* and *invalid item pruning*, plus some other optimization techniques have been newly proposed to enhance the performance. Our thorough performance study has shown that BAMBOO not only can generate orders of magnitude fewer patterns, but also can be orders of magnitude faster than both the recently developed frequent closed itemset mining algorithms and LPMiner, a frequent itemset mining algorithm with length decreasing support constraint. Furthermore, our experimental study also testifies the effectiveness of the pruning methods and good scalability of BAMBOO in terms of number of transactions.

6 Acknowledgments

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